Engaging with Mathematics

Supporting children's mathematical understanding through inclusion and interaction

Parinaz Nikfarjam, York University

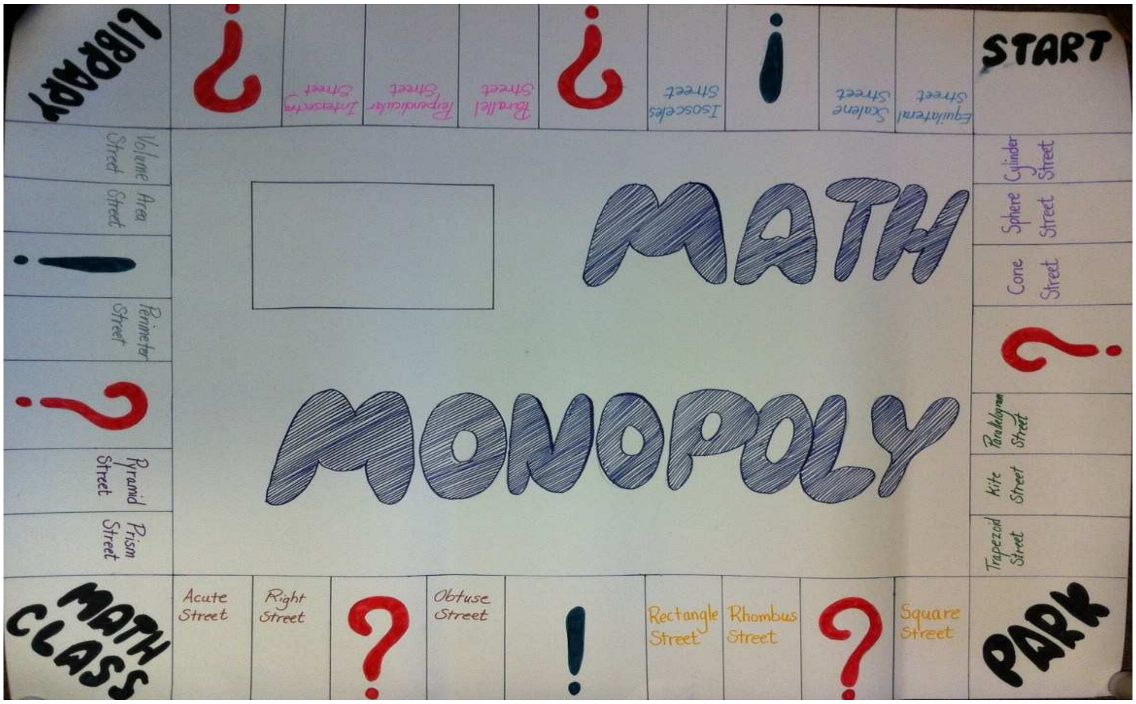
In my years of learning and teaching mathematics, I have often noticed that students' opinions toward the subject of mathematics are charged with a great deal of emotion. When asked their opinion about mathematics, students either express love or hatred for the subject. Interestingly, when examined closely, one finds that their feelings towards mathematics are often closely related to their feelings towards mathematics classrooms in general, or in many cases, a particular mathematics classroom. In other words, their feelings towards the subject of mathematics are greatly influenced by their experience in a mathematics classroom. Since students' level of engagement in mathematics is so closely related to their opinions and feelings towards the subject, it is crucial to know the underlying factors that encourage or hinder their interest.

Students who express excitement and interest towards learning and doing mathematics often have a positive image of a mathematics classroom. These students use phrases such as, "Math is so much fun", or "I'm so good at math", to describe their thoughts and feelings towards the subject. What they are in fact saying is that their experiences of math have made them feel included and empowered. As a result, these students enjoy engaging with mathematics. On the other hand, students who express dislike and even hatred toward mathematics paint a very different picture of their experiences of a mathematics classroom. They use phrases such as, "Math is so boring", or "I just don't get it" to express their frustration, disinterest, or contempt for the subject. For these students, math has never been presented in ways they can connect to, enjoy, and understand. As such, they feel disengaged, excluded, and powerless in mathematics classrooms.

As a math teacher, I have always tried to facilitate an environment that keeps students engaged, included, and empowered. Such environments are centered on students as active participants of their own learning. The following projects are two examples of teacher-created, student-centered approaches to teaching, learning, and assessing mathematics.

Project # 1: Math Monopoly

My Masters' thesis was centered on exploring the growth of early elementary mathematical understanding as students engaged in a math-based board game. One of my reasons for choosing board games was my students' increasing interest in playing different board games during class breaks. So, I decided to mix in mathematical concepts and operations with an activity they seemed to enjoy doing. Five of my own grade two and three students were participants of this research. We, my students and I, decided on the name, rules, and layout of the game, and designed and made the board, the cards, and pawns. As the game followed the general rules of a Monopoly game, my students and I decided to call it Math Monopoly.



During the course of five weeks, as students engaged in the game during and after school hours, I explored the growth of their mathematical understanding as it emerged through their interactions with one another and with the game itself. I also examined play as an assessment tool for the teacher as well as a self-assessment tool for the students. Finally, I looked at the facets of play that helped it sustain itself. The following are some of the excerpts from my research.

The Quadrilateral - A case for self-assessment and self-correction

Four students, Sarah, Sam, Mark and Adam, are engaged in play. Mark has chosen to be the banker during this game. It is Sam's turn to play. Sam rolls the die. He moves his skittle and lands on one of the prompt-card boxes. He picks a prompt card and reads it aloud. The prompt asks him to move his skittle to the street whose name is a quadrilateral with four right angles. Sam almost immediately reads the prompt again, this time reading it slowly and pausing every few words. He pauses and appears to be thinking. However, he is not looking at the board yet. He then moves forward toward the board and repeats the word "quadrilateral" to himself. He scans the board briefly and looks back at the prompt card again. This time he reads the card quietly. Adam and Sarah whisper to one another. Sam continues to scan the board. Adam asks Sam to hurry up. Sarah whispers something to Adam, and Adam says: “Yes”. Mark suggests Sam look at the question again. Adam comments that he has already done that and Mark suggests that he should do it again. Sam looks at the question, and immediately moves his skittle to one of the streets, which is not the correct answer. Adam tells him that he has chosen the wrong street. Adam asks me whether Sam gets another chance or whether others can answer the question for him. I explain that Sam gets one try only and that he can now ask someone else for the answer. Adam looks at Sam, and Sarah raises her hand. They are both waiting for Sam to pick one of them to answer the question. Sam chooses Adam. Adam holds out his hand towards Sam and asks to see the card again. He reads the question. Adam takes Sam’s skittle and places it on the Square Street. He says the street name and asks Sam for ten cents as payment. Sam takes the prompt card back from Adam to look at it again. Adam explains as he moves his hands in the air, tracing an imaginary square, showing the four sides and the right angles.

Adam: Four equal sides, four right angles.
(Sam looks at Adam, still not convinced with his explanation.)
Sam: How four equal sides?
(Adam traces a square on the board with his finger pointing to each of the imaginary right angles.)
Adam: Four equal sides *(pointing at the sides of the traced square).*
Sam: Where?
(Adam traces a square in the air.)
Adam: Square
Sam: Oh
(At the same time, Sarah picks up her pen and draws a square on her paper.)
Sarah: A square. One, two, three, four
(outlining each of the right angles with her pen as she counts).
Sarah: Four. Four right angles.
Sam: Four angles and four equal sides?
Sarah: Yeah, here.
(showing the sides of the square in her paper)
Sam: Oh, Yeah.

During the follow-up interview Sam comments about his understanding. I asked Sam, Adam, Mark, and Sarah about the square problem. The following is their response to my question.

Adam: He didn’t know. He forgot what was the quadrilateral with …
Sarah: Equal angles.
Adam: Four equal sides, and four right angles.
Sam: I thought side means faces.
Sarah: Ha?
Sam: I thought side means faces.

Mark: Shapes only have one face. It has four sides. And they are equal and they have right angles.
Sam: Sides, not faces. It has four. One, two, three, four.
(He draws an imaginary shape and shows the sides).
Sarah: And four right angles.
Sam: Yes, four right angles, four equal sides.

Playing with Multiplication: A case for interaction and collective understanding

The participants during our second game were Sarah, Kate, Adam, and Mark. During the game, Sarah's skittle landed on a prompt-card box. The card prompted Sarah to move her skittle three units backwards, four times. Sarah did not know what she should do. It was not clear whether she did not understand the wording of the question, or whether she was not capable of solving the problem. Kate showed Sarah how to do so by moving her skittle and counting up from one to three four different times. She also moved her hand up and down as she counted. Kate had decided to use repeated addition, and not multiplication, to solve this problem. As grade two and three students, all of the players were familiar with both repeated addition and multiplication. As such, during the follow-up interview I asked Kate to explain her method to me.

Parinaz: I want to know how you *(Kate)* tried to explain it to her *(Sarah)*. I remember you were using your fingers, and counting something.
Kate: Yeah.
Parinaz: Can you show us? Three… What was it?
Kate: Yeah…, like…, three times…, well I don't know how to explain it.
Adam: No, three four times.
Kate: Yeah, yeah. So, three four times. So, like, moving three four times. Kind of like, one, two, three, one. One, two, three, two. One, two, three, three. It's like that.
(She taps her fingers on the desk as she counts)
Parinaz: Ok. And you were showing it.
Mark: That's actually four times three.
Kate: Yeah.
Mark: It is much easier.
Parinaz: So, right away you know how many streets you have to move.
Kate: Yeah, twelve.
Parinaz: So, … you can use multiplication.
Mark: Or you can just do one, two, three, four, …
Sarah: Or, you can do four times three.
Kate: Yeah.
Parinaz: Would that be different? Four times three and three times four?
Sarah: No, they are different numbers. No, Different orders.
Parinaz: Different order?
Sarah: Four times three and three times four.
Adam: Three plus three plus three plus three, or four plus four plus four.

During our final interview, I asked my students to name one math-related skill they learned from a friend during the course of the games. Sarah volunteered the following response.

Sarah: I got skip count by three four times. I didn't know what it meant. She was like, "Do this." and she moved her hands.
Mark: And "Use your math."
Sarah: Oh yeah, and "Use your math."
(They all laugh.)
Parinaz: So you think her hand gesture helped you remember?
Sarah: Yes. And since then, when I see a question like that, I do it. I remember this.
(Sarah moves her hands up and down mimicking Kate's hand gestures during the game.)

What Does One-Tenth Mean? A case for assessment

According to the rules of the game the cost to purchase each street was set at two dollars (two hundred cents) and the penalty for landing on a street a player did not own was twenty cents, which is one tenth of the purchase price. During one of our interview conversations following one of the game sessions, one of the students suggested changing the purchase price for each street from two dollars to one cent. All the others were interested in this idea as they laughed and agreed to it. Except for one student (Adam), none of the others had been given lessons on decimal numbers and decimal operations. However, they had all been presented with lessons on fractions and fraction operations and it was my opinion that their understanding of fractions was going to be sufficient to at least explore this suggestion. As such, I considered this to be a great opportunity to see how they would attempt to solve the problem of calculating the penalty fee for each street. So I guided their conversation by asking, "If one street was one cent, and you bought it, and then someone else landed on that street, how much would they have to pay you?" The following excerpt is the conversation between me and the students regarding calculating the penalty fees.

Sarah: Ten cents.
Adam: No. Less than a penny.
Parinaz: What was the rule?
Adam: One-tenth of a penny.
Parinaz: One-tenth of a penny. How are you going to calculate that?
Sarah: We give nothing.
Adam: One-tenth of zero point one.
Sarah: Nothing.
Parinaz Would it be nothing?
Mark: Then it will be a quarter of a penny.
Adam: No, it is one-tenth of a penny.
Parinaz: How are we going to calculate one-tenth of a penny?

Mark: We cut it.
Sam: Yeah. Cut it to ten.
Parinaz: Do we have a coin to give for one-tenth of a penny?
Adam: No.

Project # 2: A Multi-sensory Approach to Memorizing the Multiplication Table

One of the most common difficulties my students encounter is memorizing multiplication facts. It is not just that they find it hard to keep the facts in memory, but that they find the process difficult and/or boring. I believe that most people, teachers and students in particular, approach memorization as an act of repeating facts until they are engraved in one's memory. However, we now know that people learn differently. A visual learner processes information differently from an auditory learner. By extension, the way these two memorize facts must be different. A student who learns through sensorimotor stimulation may have a hard time memorizing facts through repetition. As such, an inclusive classroom must be cognizant and supportive of different learning styles. Last year, I decided to design a multi-sensory program around memorizing multiplication facts. In this program, I planned activities that incorporate rhymes and songs, movement and gesture, music and singing, and drawing and writing. These activities were also supplemented by memorization charts and tables as well as Montessori multiplication material. In this way, memorizing multiplication facts becomes a multi-sensory circular process to which all students have access, regardless of their different learning styles.

Songs and rhymes

Writing rhymes for multiplication facts was the first step of the process. Using rhymes for the purpose of memorization is not a novel idea. Nursery rhymes and songs are amongst children's earliest experiences with language and most people remember those rhymes throughout their adulthood. I wrote eight sets of rhymes for the multiplications of two through nine. The words for each set rhyme with the number they are associated with. For instance, the end-words for multiplication of two all rhyme with the word "two". Once the lyrics were ready for each set, with the help of a friend and composer, we came up with music for each set.

Examlple: Multiplication Facts for Two

One times two
Two, that's who

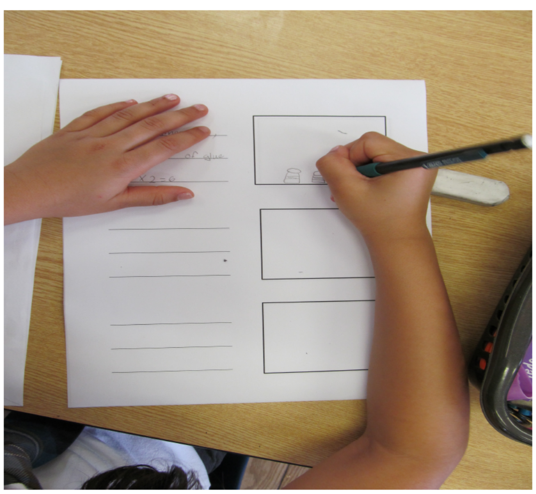
Two times two
Four shades of blue …

Movement and gestures

It was important for me to add movements and gestures to rhymes. The gestures and movements provide students with a way to associate with the rhymes, and through that, with the multiplication facts. The students decided on what gestures they wanted to do for each multiplication fact. Needless to say we had a lot of fun deciding on the gestures.

Musical instruments

Once the students were familiar with the words, they were given different musical instruments to beat to the words. We tried to associate the number of beats for every set to the number it represented. For example, when beating to the "two" multiplication song, they would beat their instruments in multiples of two in between stanzas. Gradually students would sing the songs to the music and perform the movement simultaneously.



Multiplication booklets

Once the students had plenty of experience with the words, the music, and the gestures, they made multiplication booklets for every multiplication set. Each book contains the words, the numerical representation, and a space to draw a visual representation of the multiplication.

Final Thoughts: Inclusion + Empowerment = Engagement

The Math Monopoly project, now a 4-year old project, is still being used in my mathematics classrooms. Throughout the years, students have modified it by adding new prompt cards and changing some of the rules. It has sustained its place in the classroom and has given rise to a multitude of mathematical conversations. The Multiplication project is still new and is undergoing modifications. I am always looking for new aspects to add to it as to make it more accessible to all students. Though very different in nature and process, both projects were designed to capture students' attention, facilitate their access to learning and doing mathematics, and to encourage the students' agency in their learning process. The multi-sensory project and the Math Monopoly project are both examples of how the classroom environment can facilitate a deeper engagement for the learners. Where the former serves as an inclusive environment for different learners' equal access to math, the latter provides students with a forum for interaction, collaboration, self-assessment, and empowerment.